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DESCRIPTION OF COMPUTER PROGRAM CAB3DYN FOR THE TIME DOMAIN ANALYSIS OF THE THREE-DIMENSIONAL LONG-TIME DEPLOYMENT BEHAVIOR OF GENERAL OCEAN CABLE SYSTEMS

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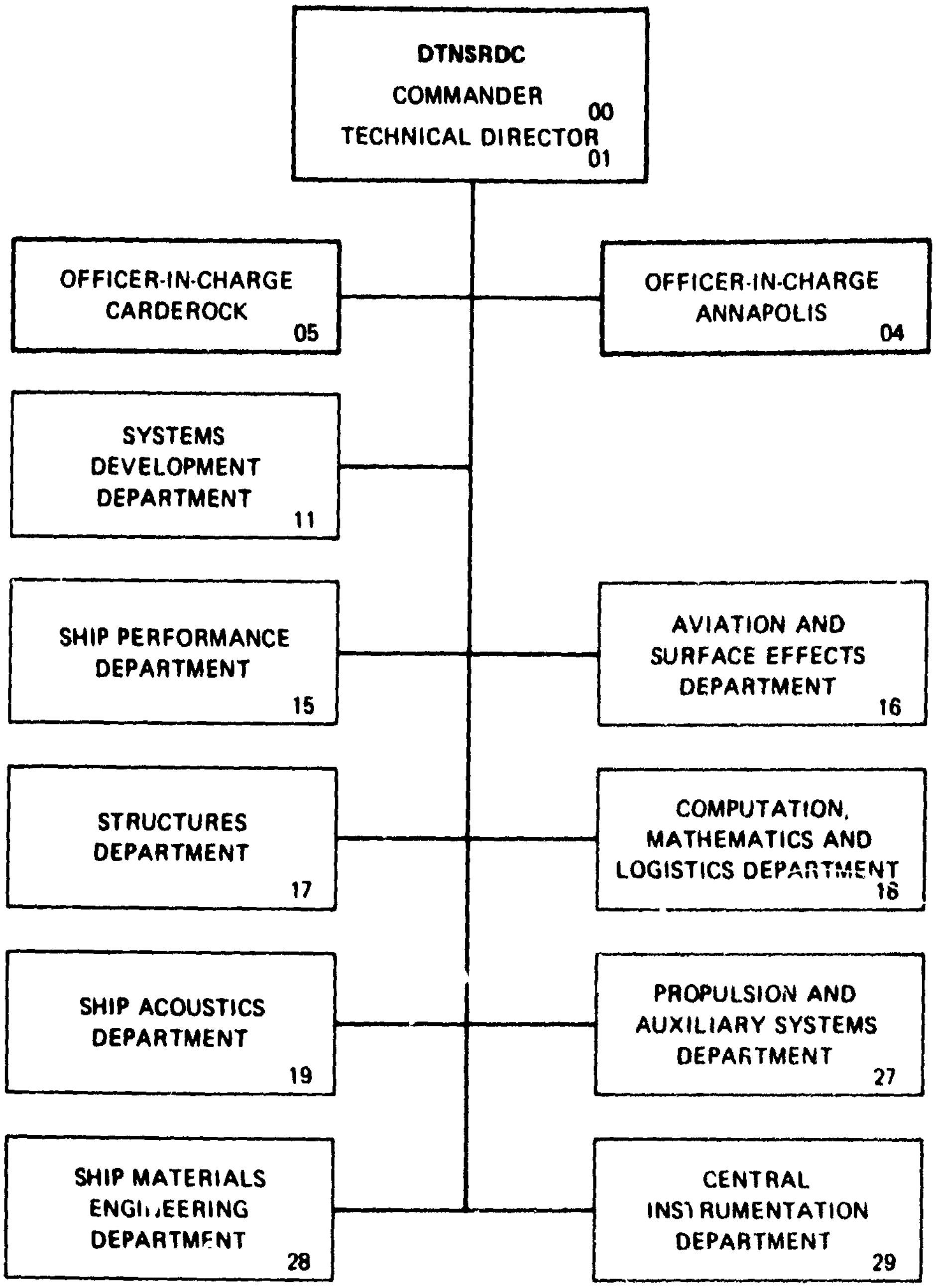

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ABSTRACT

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INTRODUCTION

This report presents a detailed description of Program CAB3DYN, which analyzes in the time domain the three-dimensional long-time deployment behavior of free-floating and towed ocean cable systems. The term long-time dynamic behavior refers to motions where time scales are large enough so that inertia effects are not of major importance. Examples of such motions are the deployment of a free-floating cable system from an arbitrary initial state to the final steady-state and the behavior of a towing cable undergoing a prescribed maneuver.

To some extent, the formulation in Program CAB3DYN may be considered to be an extension of the two-dimensional dynamic formulation contained in Program CABMOD [1,2] to three dimensions by using the coordinate system of the three-dimensional steady-state Program CAB3E [3,4]. The cable system consists of a buoy or towing platform at the surface, a cable whose properties may vary along its length, and the presence of intermediate bodies along the cable. The current profile is modeled by a series of input magnitudes and directions at various depths, as in Program CAB3E. The cable is represented by a series of straight segments, as in Program CABMOD.

However, there are significant differences. Those factors which are perturbations to the overall mean long-time motion are not considered in the present program. In particular, ocean waves and rotational modes of the surface buoy are neglected. These result in considerable simplification of the formulation and reduction in the number of input variables for the surface buoy. On the other hand, hydrostatic buoyancy and drag characteristics of the buoy are modeled in somewhat greater detail. Another considerable simplification results from the neglect of added inertia for the cable. This eliminates coupling between the equations of motion for a given cable node. For two sample problems it is shown that the cable added inertia terms have nearly negigible effect on the motion.

The report starts by giving a brief review of cable studies which are most closely related to the present study. The report then describes the coordinate systems used in the program, the modeling of the time-varying current profile, and the formulations for the surface buoy (or prescribed surface motion), intermediate bodies, and the connecting cable. A description is given of the various subroutines of the program. Detailed input instructions are given, including a listing of the input READ statements, the definition of each input variable, and

comments on the entry of input data. Wherever possible, the input variables are matched to those contained in Program CABMOD.

Two sample problems are given to illustrate usage of the program, output, and computer execution time. The two problems involve the deployment of a free-floating buoy-cable system and a towing cable from an initially vertical position to the steady-state configuration in the presence of various current profiles and towing speeds, respectively. For each problem, results computed by Program CAB3DYN are compared to those obtained by Program CABMOD for two-dimensional cases. It is shown that when the option of neglecting cable added inertia terms in Program CABMOD is exercised, the two programs agree identically for the towing cases and agree in an average sense for the free-floating cases. The presence of the buoy pitch terms in Program CABMOD introduces a perturbation to the overall motion which is not modeled by Program CAB3DYN. For both problems, it is further shown that there is little difference between the motions predicted by Program CABMOD with and without cable added inertia effects.

For the free-floating buoy-cable problem, an efficient approach is indicated for using Program CAB3DYN, in conjunction with Program CAB3E, to predict the detailed configuration of the system in the presence of a slowly varying current profile. For the towing cable problem, results calculated by Programs CAB3DYN and CABMOD are compared to those calculated by Program CABTRAN[5], which is a two-dimensional program which completely neglects inertia effects. It is shown that for cases where the cable is towing a neutrally buoyant lower array, Programs CABMOD and CAB3DYN predict unstable motions of the lower segment at higher towing speeds while Program CABTRAN predicts incorrect steady-state configurations. For cases of a cable alone, with no lower array, the results predicted by Programs

CABMOD and CAB3DYN agree reasonably well with those predicted by Program CABTRAN.

The report concludes by giving a brief summary of the results for the two sample problems.

CALCULATION OF CABLE TENSION

PREVIOUS APPROACHES

It is well known that for most cable systems, computer time and cost are determined by the need to track the high-frequency longitudinal vibrational mode along the cable. In particular, the maximum time step Δt_M which may be taken is given by

$$\Delta t_M \propto \sqrt{\mu L/C_l} \qquad (1)$$

where μ is the mass per unit length of the cable

L is the length of the cable segment

C_l (= AE) is the elastic constant of the cable

A is the cross-sectional area of the cable

E is the elastic modulus

For transient or other high-frequency motions, it is of interest to carefully model the longitudinal vibrations. However, for the present case of deployment motions, whose time scales are orders of magnitude longer than that of the longitudinal vibrations, it is of interest to avoid the time-step restriction imposed by Equation (1). Thus, previous studies which consider this aspect were carefully investigated.

For the most part, these studies attempt to increase the allowable time step by using the condition of inextensibility to obtain the tension, or avoid the tension calculation altogether by considering only motions normal to the cable.

Walton and Polachek[6] add to the equations of motion for the cable additional nonlinear equations on the displacement which express the condition of inextensibility. At each time step, the tensions along the cable are iterated a number of times until the inextensibility condition is satisfied. Froidevaux and Scholten[7] also use an iterative procedure to calculate the tension. Their procedure consists of continually guessing the tension at the top of the cable and calculating the tension along the cable by taking the longitudinal motion to be the same along the cable until the calculated tension below the last body is nearly zero.

Ketchman and Lou[8] show that for cases where the surface motion is prescribed and the lower body is cable-like, such as an array, the tension can be directly calculated along the entire cable, starting from the lower body. However, for cases where the lower body, or any body along the cable, is not cable-like (i.e., has drag and added inertia properties which depend on fixed spatial directions rather than on cable inclination) the calculation of tension would again not be straightforward. Similar complexities would arise if the surface motion is not prescribed, as in the case of a surface buoy.

Rupe and Thresher[9] and Paul and Soler[5], use methods which solve for only the motions normal to the cable and thus avoid the need to calculate the tension. After the normal motions have been calculated, the tension may be obtained, if desired. Rupe and Thresher consider the cable to be a compound pendulum composed of a series

of links. They solve for the angular coordinates by using Lagrange's Method. Paul and Soler use a unique approach. They show that for typical towing maneuvers, it is permissible to neglect all inertia effects, thus reducing the equations of motion to first order. They then show that it is possible to start at the bottom link and use a quasi-static approach which balances drag and weight forces to obtain the normal velocities along the entire cable. However, as in the case of Reference 8, both of the above approaches work well for only the case where the surface motion is prescribed and the cable is free of bodies or has only cable-like bodies. The presence of non-cable-like bodies would make their formulations more complex and the presence f a surface buoy, whose motion is not prescribed, would make the calculation of tension an essential part of the procedure.

PRESENT APPROACH

In view of the iteration procedure required in References 6 and 7, and the lack of generality of the approaches given in References 5, 8, and 9, it was decided to take the tension T to be proportional to the strain ϵ , as follows:

$$T = T_r + C_1 \epsilon \tag{2}$$

where T_r is the reference tension at which the strain is taken to be zero. In Program CABMOD, there is an internal damping term proportional to strain rate $\dot{\epsilon}$, and ϵ is raised to an arbitrary power C_2. Computer cost may be minimized by using the reduced modulus technique[10] introduced by the first author. The technique essentially maximizes the allowable time step Δt_M

given in Equation (1) by replacing the actual elastic constant C_1 by the lowest permissible value which will still give accurate results. For deployment problems, it appears reasonable to use values of C_1 which will lead to maximum strains of 0.01. A one percent elongation should have little effect on the transverse deployment motion.

COORDINATE SYSTEMS

The two coordinate systems used in the computer program are shown in Figure 1. As mentioned in the Introduction, these coordinate systems are identical to those used in Program CAB3E[4].

The equations of motion for the cable system are derived for the spatial (x, y, z) coordinate system whose origin lies at the undisturbed ocean surface. The x and z axes lie in the horizontal plane and the y axis is positive in the direction of gravity. Cable drag is most conveniently derived for a second coordinate system attached to the cable, (X, Y, Z), where the Y-axis is directed along the cable. The cable coordinate system is obtained by first rotating the spatial system by the angle ϕ_V about the z axis and then rotating the intermediate (x^1, y^1, z^1) system by the angle θ about the x^1 axis.

The direction cosines between the spatial and cable coordinate systems are obtained as the elements of the square matrix $[A]$, given by the product of the elementary transformation matrices $[\theta]$ and $[\phi_V]$

$$[A] = [\theta][\phi_V]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi_V & \sin\phi_V & 0 \\ -\sin\phi_V & \cos\phi_V & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_V & \sin\phi_V & 0 \\ -\cos\theta\sin\phi_V & \cos\theta\cos\phi_V & \sin\theta \\ \sin\theta\sin\phi_V & -\sin\theta\cos\phi_V & \cos\theta \end{bmatrix} \quad (3)$$

These direction cosines are also shown in Figure 1. The matrix $[A]$ is used to express the components of a vector given in the spatial (x, y, z) coordinate system in terms of the cable (X, Y, Z) system, while the matrix $[A^{-1}]$ is used to express the components of a vector given in the cable (X, Y, Z) systems in terms of the spatial (x, y, z) system. Since the transformation matrix $[A]$ is orthogonal, the inverse of $[A]$, $[A^{-1}]$, is simply the transpose of $[A]$, $[A^T]$

$$[A^{-1}] = [A^T] \tag{4}$$

CURRENT PROFILE

As in Program CAB3E[4], the current profile is restricted to components in the horizontal x and z directions, which is the usual case for ocean current profiles. However, the formulation in Program CAB3E is generalized to allow the current magnitude c and angle with the x-axis, α, to have steady-state as well as time-varying components, as follows

$$c(y,t) = c_0(y) + c_1(y) \sin(2\pi f_c t) \tag{5}$$

$$\alpha(y,t) = \alpha_0(y) + \alpha_1(y) \sin(2\pi f_c t) \tag{6}$$

where the subscripts o and 1 denote steady-state and time-varying components, respectively, f_c is the frequency of the current variation

t is the time

The x and z components of the current, c_x and c_z, are given by

$$c_x = c \cos \alpha \tag{7}$$

$$c_z = c \sin \alpha \tag{8}$$

PRESCRIBED SURFACE MOTIONS

As in the case of the Program CABMOD, the program allows the user either to prescribe the motion at the surface, as in a towing maneuver case, or describe the motion by means of differential equations for a surface buoy.

The prescribed motion may be composed of a series of sinusoidal components in (a) the x and y directions, or (b) the x and z directions.

(a)

$$x_s = \sum_{i=1}^{N} a_{xi} \cos(-2\pi f_i t + \phi_i) \tag{9a}$$

$$y_s = \sum_{i=1}^{N} -a_{yi} \sin(-2\pi f_i t + \phi_i) \tag{9b}$$

(b)

$$x_s = \sum_{i=1}^{N} a_{xi} \cos(-2\pi f_i t + \phi_i) \tag{10a}$$

$$z_s = \sum_{i=1}^{N} -a_{zi} \sin(-2\pi f_i t + \phi_i) \tag{10b}$$

where the subscript s denotes the surface

a_{xi}, a_{yi}, and a_{zi} are the amplitudes of the ith sinusoidal component in the x, y, and z directions, respectively.

f_i and ϕ_i are the frequency and phase angle of the ith sinusoidal component, respectively.

Prescribed motion (a) may represent a surface ship in waves while prescribed motion (b) may represent a circular towing case.

The user may make use of the subroutine PRESC to prescribe more general towing maneuvers. For example, a build up in towing speed V from 0 to 10 knots may be prescribed according to the formula

$$V = 10 \times 1.6878 \tanh (t/200) \tag{11}$$

SURFACE BUOY FORCES

As mentioned in the Introduction, surface waves and rotational modes of the buoy are considered to be perturbations to the overall mean motion and have been neglected. This leads to a number of considerable simplifications. First, the exciting forces due to surface waves need not be computed. Secondly, equations of motion need not be written for the rotational modes, thus also eliminating the coupling between the rotational and translational modes. Thirdly, locations of various forces need not be computed or specified.

For the sake of convenience, it is further assumed that the buoy is axisymmetric so that its added inertia and drag area are the same for motion in any horizontal direction. This is the usual case for the surface buoy of free-floating cable systems. Under the above assumptions, the resultant forces acting on the surface buoy are as follows:

$$\vec{I}_S = M_{VSH}\ddot{x}\vec{i} + M_{VSy}\ddot{y}\vec{j} + M_{VSH}\ddot{z}\vec{k} \tag{12}$$

$$\vec{D}_S = \tfrac{1}{2}\rho\left(C_D A_{SH}\sqrt{\dot{x}_r^2+\dot{z}_r^2}\;\dot{x}_r\vec{i} + C_D A_{Sy}\,\dot{y}_r|\dot{y}_r|\,\vec{j} + C_D A_{SH}\sqrt{\dot{x}_r^2+\dot{z}_r^2}\;\dot{z}_r\vec{k}\right) \tag{13}$$

$$\vec{G}_S = W_{SA}\vec{j} \tag{14}$$

$$\vec{B}_S = -\rho g V\vec{j} \tag{15}$$

$$\overrightarrow{WF}_S = WF_x\vec{i} + WF_z\vec{k} \tag{16}$$

$$\vec{T}_b = T_{bx}\vec{i} + T_{bz}\vec{k} \tag{17}$$

where the meanings of the variables are as follows: I, D, G, B, WF, T = (inertia, drag, gravity, buoyancy, wind, and tension forces, respectively), $\vec{i}$, $\vec{j}$, and $\vec{k}$ = unit vectors in the x, y, and z directions, respectively.

M_{SVH}, M_{SVy} = virtual mass in the horizontal and vertical directions, respectively,

ρ = fluid density

C_D = drag coefficient

A_{SH}, A_{Sy} = drag areas in the horizontal and vertical directions, respectively,

$\dot{x}_r = -\dot{x} + C_x$

$\dot{y}_r = -\dot{y}$

$\dot{z}_r = -\dot{z}$

W_{SA} = weight in air,

g = gravity constant,

V = submerged volume,

the subscripts S and b denote the surface buoy and the tension force acting below the buoy, respectively, and a dot denotes a derivative with respect to time.

INTERMEDIATE BODY FORCES

The forces acting on the bodies placed along the cable are somewhat similar to those previously derived for the surface buoy. The principal differences occur due to the fact that the bodies have a constant submerged volume, experience no wind force and have cable tension forces acting above and below. In the case of the lower body, a special provision is made for a time-varying thrust which may serve to alter the configuration of the cable system. The resultant forces are as follows.

$$\vec{I}_B = M_{BVH}\ddot{x}\vec{i} + M_{BVy}\ddot{y}\vec{j} + M_{BVH}\ddot{z}\vec{k} \tag{18}$$

$$\vec{D}_B = \frac{1}{2}\rho\sqrt{\dot{x}_r^2+\dot{y}_r^2+\dot{z}_r^2}\left(C_DA_{BH}\dot{x}_r\vec{i} + C_DA_{By}\dot{y}_r\vec{j} + C_DA_{BH}\dot{z}_r\vec{k}\right) \tag{19a}$$

$$0.5 \le \frac{C_DA_{BH}}{C_DA_{By}} \le 2$$

$$\vec{D}_B = \frac{1}{2}\rho\left(C_DA_{BH}\sqrt{\dot{x}_r^2+\dot{z}_r^2}\,\dot{x}_r\vec{i} + C_DA_{By}\dot{y}_r|\dot{y}_r|\vec{j} + C_DA_{BH}\sqrt{\dot{x}_r^2+\dot{z}_r^2}\,\dot{z}_r\vec{k}\right), \quad \frac{C_DA_{BH}}{C_DA_{By}} < 0.5 \text{ or } > 2 \tag{19b}$$

$$\vec{G}_B + \vec{B}_B = W_B\vec{j} \tag{20}$$

$$\vec{T}_a + \vec{T}_b = (T_{ax}+T_{bx})\vec{i} + (T_{ay}+T_{by})\vec{j} + (T_{az}+T_{bz})\vec{k} \tag{21}$$

Lower body only:

$$\vec{BT} = BT\left[\cos(2\pi f_T t + \phi_T)\vec{i} + \sin(2\pi f_T t + \phi_T)\vec{k}\right] \tag{22}$$

where the subscript B denotes the body, the subscripts a and b refer to above and below the body, respectively, W_B is the weight in fluid of the body, BT, f_T, and ϕ_T are the magnitude, frequency, and phase of the time varying force acting on the lower body.

The formulation for drag force given in Equation (19a) is appropriate for a blunt shape such as a cube or sphere. The formulation given in Equation (19b) is more appropriate for a cylinder or disk shape.

CABLE FORCES

Following Program CABMOD, the continuous cable is divided into a number of massless straight elastic segments, as shown in Figure 2. All the cable forces are equally divided between the two nodes at the ends of the segment. The nodes may represent convenient locations to represent cable shape or may correspond to locations of bodies placed along the cable, including the surface buoy and lower body.

The formulation for cable forces is similar to the formulation in Program CABMOD with three principal differences. First, a somewhat simpler tension-strain relation is used, as given by Equation (2). Secondly, the formulation is extended to three dimensions using the coordinate systems shown in Figure 1. Thirdly, added inertia terms for the cable are neglected. This not only reduces the number of terms but also eliminates coupling of the equations of motion for a given node.

The forces are given for a cable segment of length ℓ. Recall that these forces must be equally divided between the nodes at the ends of the segment.

The two simplest forces which require no coordinate transformation are the inertia force $\vec{I}_c$ and weight-in-fluid force $\vec{W}_c$

$$\vec{I}_c = \mu\ell\,(\ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + \ddot{z}\,\vec{k}) \tag{23}$$

$$\vec{W}_c = w l \vec{j} \tag{24}$$

where the subscript c denotes the cable

μ is the mass per unit length

w is the weight per unit length in fluid

The tension and drag forces require coordinate transformations and will be discussed separately. The tension force T is given in Equation (2) as

$$T = T_r + C_1 \epsilon \tag{2}$$

The strain ϵ is given by

$$\epsilon = \frac{\Delta l}{l_r} = \frac{l}{l_r} - 1 = \frac{\sqrt{(x_l - x_u)^2 + (y_l - y_u)^2 + (z_l - z_u)^2}}{l_r} - 1 \tag{25}$$

where l_r is the cable length at $T = T_r$

Δl is the change in length

the subscripts l and u refer to the lower and upper ends of the cable segment, respectively

The components of the tension in the spatial x, y, and z directions are obtained by using the matrix $[A^{-1}]$, given by Equations (3) and (4), and noting that the tension force for the case where the cable is below the node, T_h, is directed in the positive Y direction.

$$\begin{bmatrix} T_{bx} \\ T_{by} \\ T_{bz} \end{bmatrix} = \left[A^{-1}\right] \begin{bmatrix} 0 \\ T_b \\ 0 \end{bmatrix} \tag{26}$$

which leads to

$$T_{bx} = -\cos\theta_b \sin\phi_{Vb} T_b \tag{27a}$$

$$T_{by} = \cos\theta_b \cos\phi_{Vb} T_b \tag{27b}$$

$$T_{bz} = \sin\theta_b T_b \tag{27c}$$

For the case where the cable is above the node such that the tension force T_a is directed in the negative Y direction, the components in the spatial directions are obtained by simply reversing the signs in the above equation

$$T_{ax} = +\cos\theta_a \sin\phi_{Va} T_a \tag{28a}$$

$$T_{ay} = -\cos\theta_a \cos\phi_{Va} T_a \tag{28b}$$

$$T_{az} = -\sin\theta_a T_a \tag{28c}$$

The angles ϕ_V and θ appearing in the above equations may be obtained by considering the following three direction cosines from Figure 1.

$$\cos(x, Y) = \frac{\Delta x}{\Delta \ell} = -\cos\theta \sin\phi_V \tag{29a}$$

$$\cos\,(y,\,Y) = \frac{\Delta y}{\Delta \ell} = \cos\theta\,\cos\phi_V \tag{29b}$$

$$\cos\,(z,\,Y) = \frac{\Delta z}{\Delta \ell} = \sin\theta \tag{29c}$$

Equation (29a) divided by Equation (29b) yields the following equation for ϕ_V

$$\phi_V = \tan^{-1}\left(\frac{-\Delta x}{\Delta y}\right) = \tan^{-1}\left[\frac{-(x_\ell - x_u)}{(y_\ell - y_u)}\right] \tag{30}$$

while Equation (29c) divided by Equation (29b) yields the equation for θ

$$\theta = \tan^{-1}\cos\phi_V\,\frac{\Delta z}{\Delta y} = \tan^{-1}\cos\phi_V\,\frac{(z_\ell - z_u)}{(y_\ell - y_u)} \tag{31}$$

The normal and tangential drag forces are taken to be respectively proportional to the squares of the normal and tangential components of the fluid velocity relative to the cable $\vec{v}_r$ given by

$$\vec{v}_r = -\vec{v}_c + \vec{c} = \dot{x}_r\,\vec{i} + \dot{y}_r\,\vec{j} + \dot{z}_r\,\vec{k} \tag{32}$$

where

$$\dot{x}_r = -\dot{x} + c_x$$

$$\dot{y}_r = -\dot{y}$$

$$\dot{z}_r = -\dot{z}$$

c_x and c_z are respectively the x and z components of current given by Equations (5) to (8).

The components of $\vec{v}_r$ in the cable X, Y, and Z directions are obtained by using the matrix $[A]$, Equation (3)

$$\begin{bmatrix} v_{rX} \\ v_{rY} \\ v_{rZ} \end{bmatrix} = [A] \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{z}_r \end{bmatrix} \tag{33}$$

which leads to

$$v_{rX} = \cos\phi_V \dot{x}_r + \sin\phi_V \dot{y}_r \tag{34a}$$

$$v_{rY} = -\cos\theta \sin\phi_V \dot{x}_r + \cos\theta \cos\phi_V \dot{y}_r + \sin\theta \dot{z}_r \tag{34b}$$

$$v_{rZ} = \sin\theta \sin\phi_V \dot{x}_r - \sin\theta \cos\phi_V \dot{y}_r + \cos\theta \dot{z}_r \tag{34c}$$

The drag forces D_{CX}, D_{CY}, and D_{CZ} in the X, Y, and Z directions are given by

$$D_{CX} = \frac{1}{2}\rho C_D d\ell v_{rX} \sqrt{v_{rX}^2 + v_{rZ}^2} \tag{35a}$$

$$D_{CY} = \frac{1}{2}\rho C_T d\ell v_{rY} |v_{rY}| \tag{35b}$$

$$D_{CZ} = \frac{1}{2}\rho C_D d\ell v_{rZ} \sqrt{v_{rX}^2 + v_{rZ}^2} \tag{35c}$$

where d is the cable diameter

C_D is the normal drag coefficient

C_T is the tangential drag coefficient

In order to use these forces in the equations of motion it is necessary to express them in terms of the spatial directions by using the matrix $\left[A^{-1}\right]$

$$\begin{bmatrix} D_{Cx} \\ D_{Cy} \\ D_{Cz} \end{bmatrix} = \left[A^{-1}\right] \begin{bmatrix} D_{CX} \\ D_{CY} \\ D_{CZ} \end{bmatrix} \tag{36}$$

which leads to

$$D_{Cx} = \cos\phi_V D_{CX} - \cos\theta \sin\phi_V D_{CY} + \sin\theta \sin\phi_V D_{CZ} \tag{37a}$$

$$D_{Cy} = \sin\phi_V D_{CX} + \cos\theta \cos\phi_V D_{CY} - \sin\theta \cos\phi_V D_{CZ} \tag{37b}$$

$$D_{Cz} = \sin\theta D_{CY} + \cos\theta D_{CZ} \tag{37c}$$

FINAL EQUATIONS OF MOTION

For an intermediate node, the final equations of motion are obtained by using the body forces given by Equations (18) to (21) and adding to them the contributions from the forces of the cable segments above and below the node, Equations (23) to (37), resulting in

$$\begin{aligned} \vec{I}_B + \tfrac{1}{2}(\vec{I}_{Ca} + \vec{I}_{Cb}) &= \left[W_B + \tfrac{1}{2}(w_a l_a + w_b l_b)\right]\vec{j} \\ &+ \left[\vec{D}_B + \tfrac{1}{2}(\vec{D}_{Ca} + \vec{D}_{Cb})\right] + \left[\vec{T}_a + \vec{T}_b\right] \end{aligned} \tag{38}$$

For a general three-dimensional case, the above vector equation leads to three uncoupled scalar second-order differential equations for the x, y, and z locations of the node.

For the lower body, the quantities involving the subscript b are equal to zero since there is no cable below the body. In addition, there is a thrust force BT acting in the x and z directions.

For the surface buoy, the vector equation of motion is obtained by using the buoy forces given by Equations (12) to (17), and adding to them the contributions from the forces of the cable segment below the buoy, Equations (23) to (37), resulting in

$$\left(\vec{I}_S + \frac{1}{2}\vec{I}_{Cb}\right) = \left(W_{SA} - \rho g V + \frac{1}{2} w_b \ell_b\right)\vec{j} + \left(\vec{D}_S + \frac{1}{2}\vec{D}_{Cb}\right) + \overrightarrow{WF}_S + \vec{T}_b \qquad (39)$$

DESCRIPTION OF COMPUTER PROGRAM

Program CAB3DYN consists of a main program and five subroutines.

MAIN PROGRAM

The main program accepts input data for the cable system, current profile, initial conditions, and data for either a surface buoy or prescribed motions according to Equations (9) or (10). Data may be entered in either English or metric units. A detailed description of input directions is given in the next chapter.

The program is currently written to accept up to 25 cable segments and intermediate bodies. This number can be easily increased by changing a few DIMENSION

and COMMON statements, but it should be noted that calculations for more than 25 nodes are likely to require prohibitively large amounts of computer time.

The main program prints out the input data, and then calls on the subroutine KUTMER to calculate the motions of the system at prescribed time intervals. The main program then prints out the calculated displacements, velocities, accelerations, and tensions.

SUBROUTINE DYNA

This subroutine defines the vector differential equation of motion (38) for each of the N nodes. The subroutine also defines the vector differential equation of motion (39) for the case of a surface buoy, or the prescribed motion according to Equations (9) or (10). In the case of a surface buoy, this subroutine calls the subroutine XLINEAR to furnish the values of buoy volume and drag area as a function of buoy submergence. In the case of a prescribed motion not covered by Equations (9) or (10), the subroutine calls the subroutine PRESC for the motions.

SUBROUTINE CUR

This subroutine furnishes the x and z components of the time-varying current profile. For a given value of the vertical distance y, the subroutine linearly interpolate between the quantities c_0 , α_0 , c_1 , and α_1, defined in Equations (5) and (6), which are input as a function of y. The subroutine then calculates the x and z components of current according to Equations (7) and (8). For cases where the given value of y is greater (less) than the largest (smallest) value of y which is read in, the subroutine takes the values of c_0 , α_0, c_1 , and α_1 , to be the values at the largest (smallest) value of y which is read in.

SUBROUTINE KUTMER

This subroutine uses the Kutta-Merson method to numerically integrate the differential equations of motion defined in subroutine DYNA. The subroutine automatically reduces the integration step size until the specified error criteron of 0.0001 is satisfied.

SUBROUTINE XLINEAR

This subroutine furnishes the volume V and drag area $C_D A_{SH}$ as a function of buoy submergence H. The subroutine linearly interpolates between the input values of volume and drag area which are read in as a function of submergence. For cases where the given submergence is greater (less) than the largest (smallest) value of submergence which is read in, the subroutine takes the volume and drag area to be the value at the largest (smallest) value of submergence which is read in.

SUBROUTINE PRESC

The user may use this subroutine to prescribe an arbitrary motion which is outside the form covered by Equations (9) and (10).

PROGRAM STORAGE AND TIME REQUIREMENTS

On the CDC 6700 computer currently in use at the Center, Program CAB3DYN requires a memory of approximately 41,000 octal words to load, 30,400 octal words to execute, and approximately 18 seconds to compile. By comparison, Program CABMOD requires a memory of approximately 52,500 octal words to load, 41,000 octal words to execute, and approximately 30 seconds to compile. Thus, Program CAB3DYN is a simple program, despite its three-dimensional capability. This

is largely due to the previously mentioned neglect of surface waves, cable added inertia, and buoy rotational modes.

As in the case of Program CABMOD, execution time depends largely on the value of the elastic constant C_1, the number of cable nodes, and the amount amount of time over which dynamic motions are desired. For the sample problems given in this report, execution time for Program CAB3DYN ranges from 1.2 to 2.5 times the corresponding time for Program CABMOD. In these problems, the ratio of execution time to real time, ET/RT, ranges from 0.6 to 3.3.

INPUT INSTRUCTIONS

READ STATEMENTS

Input data are entered into the program by means of the following READ statements in the MAIN program. Wherever possible, the names and definitions of input variables are matched to those contained in Program CABMOD.

```
      READ(5,1) NCASES
      DO 1000 MC=1,NCASES
      READ(5,301) TITLE
  301 FORMAT(20A4)
      READ(5,1) NSM,NCAB,NCUR,MTRC,ISM
      READ(5,2) (FSM(K),K=1,NSM)
    2 FORMAT(8F10.4)
    3 FORMAT(8F10.2)
    4 FORMAT(8F10.6)
    5 FORMAT(8F10.0)
      READ(5,2) (AXSM(K),K=1,NSM)
      READ(5,2) (AYSM(K),K=1,NSM)
      READ(5,2) (AZSM(K),K=1,NSM)
      READ(5,2) (FIDSM(K),K=1,NSM)
      READ(5,2) RHO,TMIN,CCFR
      READ(5,2) BTM,BTF,BTPD
      READ(5,2) TINV1,DT1,TOTT,DT2,TBYMX
      READ(5,3) (FLC(K),K=1,NCAB)
      READ(5,2) (DCI(K),K=1,NCAB)
      READ(5,2) (CDN(K),K=1,NCAB)
      READ(5,2) (CDT(K),K=1,NCAB)
      READ(5,2) (WC(K),K=1,NCAB)
```

```
      READ(5,4) (CM(K),K=1,NCAB)
      READ(5,3) (TREF(K),K=1,NCAB)
      READ(5,5) (C1(K),K=1,NCAB)
      READ(5,2) (WBD(K),K=1,NCAB)
      READ(5,2) (CDABX(K),K=1,NCAB)
      READ(5,2) (CDABY(K),K=1,NCAB)
      READ(5,2) (XMBV(K),K=1,NCAB)
      READ(5,2) (YMBV(K),K=1,NCAB)
      READ(5,3) (YY(I),I=1,NCUR)
      READ(5,3) (CCZK(K),K=1,NCUR)
      READ(5,3) (CC1K(K),K=1,NCUR)
      READ(5,3) (CCZAD(K),K=1,NCUR)
      READ(5,3) (CC1AD(K),K=1,NCUR)
      READ(5,2) (PHID(I),I=1,NCAB)
      READ(5,2) (THETAD(I),I=1,NCAB)
      READ(5,3) (TENI(I),I=1,NCAB)
      READ(5,2) (XPI(I),I=1,NCAB)
      READ(5,2) (YPI(I),I=1,NCAB)
      READ(5,2) (ZPI(I),I=1,NCAB)
      IF (FSM(1).LE.2000.) GO TO 220
      READ(5,2) XSI,YSI,ZSI,XPSI,YPSI,ZPSI,WFM,WFAD
      READ(5,2) CDASBY,WASB,XMSBV,YMSBV,RTX,RTY,RTZ,DFTLIM
  220 CONTINUE
```

The corresponding FORMAT statements are:

```
  1    FORMAT (24I3)
  2    FORMAT (8F10.4)
  3    FORMAT (8F10.2)
  4    FORMAT (8F10.6)
  5    FORMAT (8F10.0)
301    FORMAT (20A4)
```

DEFINITION OF INPUT VARIABLES

NCASES	Number of cases, NCASES $\geq$ 1
TITLE	Title
NSM [1] *	Number of surface motion components, 1 $\leq$ NSM $\leq$ 20
NCAB	Number of cable segments, 2 $\leq$ NCAB $\leq$ 25

---

* Superscripts refer to explanatory notes which begin on page 27

NCUR	Number of current profile points, $2 \leq$ NCUR ≤ 10
MTRC	MTRC ≤ 0 if input data are enetered in English units; MTRC ≥ 1 if input data are entered in metric units**
ISM	ISM ≤ 0 if surface motions are prescribed in x and y directions, Equation (9); ISM ≥ 1 if surface motions are prescribed in x and z directions, Equation (10)
FSM(K)[1] AXSM(K)[1]	$x_{SM} = \sum_{k=1}^{NSM} AXSM(K) * \cos(-2\pi * FSM(K) * t + FIDSM(K) * \pi/180.)$
AYSM(K)[1] AZSM(K)[1]	$y_{SM} = \sum_{k=1}^{NSM} -AYSM(K) * \sin(-2\pi * FSM(K) * t + FIDSM(K) * \pi/180.)$
FIDSM(K)[1]	$z_{SM} = \sum_{k=1}^{NSM} -AZSM(K) * \sin(-2\pi * FSM(K) * t + FIDSM(K) * \pi/180.$
RHO	Fluid density in slugs/feet^3 or kilograms /meters^3
TMIN	Minimum tension which can be supported by cable in pounds or Newtons; TMIN = 0 for a flexible cable
CCFR	Frequency of current variation in cycles per second
BTM, BTF, BTPD	Bottom thrust $\vec{BT} = BTM * [\cos(2\pi * BTF * t + BTPD * \pi/180.)\,\vec{i} + \sin(2\pi * BTF * t + BTPD * \pi/180.)\,\vec{k}]$
TINV1	Initial time interval in seconds for motion calculations
DT1	Time step in seconds for which printout is desired $0 \leq t \leq$ TINV1
TOTT	Total time in seconds for which motion calculations are desired
DT2	Time step in seconds for which printout is desired for TINV1 $\leq t \leq$ TOTT

** All data must be input in consistent units.

TBYMX	Maximum absolute value of tension in cable just below buoy in pounds or Newtons
FLC(K)	Length of Kth cable segment in feet or meters
DCI(K)	Diameter of Kth cable segment in inches or centimeters
CDN(K)	Normal drag coefficient of Kth cable segment
CDT(K)	Tangential drag coefficient of Kth cable segment
WC(K)	Weight in fluid of Kth cable segment in pounds/foot or Newtons/meter
CM(K)	Mass of Kth cable segment in slugs/foot or kilograms/meter
TREF(K)	Reference tension of Kth cable segment in pounds or Newtons
C1(K)	Elastic constant of Kth cable segment in pounds or Newtons
WBD(K)	Weight in fluid of Kth body in pounds or Newtons
CDABX(K), CDABY(K)	Drag area of Kth body in feet^2 or meters^2 for flow in (horizontal,vertical) directions
XMBV(K) YMBV(K)	Virtual mass (mass + added mass) of Kth body in slugs or kilograms for motion in (horizontal,vertical) directions
YY(I)	Value of y in feet or meters
CCZK(K) CC1K(K)	Magnitudes (c_0 , c_1) of (steady, time-varying) component of current in knots or meters/second at y = YY(K), see Equations (5) to (8)
CCZAD(K) CC1AD(K)	Angles (α_0, α_1) measured from x-axis of (steady, time-varying) component of current in degrees at y = YY(K), see Equations (5) to (8)
PHID(I)	Initial value of ϕ_V of Ith cable segment in degrees
THETAD(I)	Initial value of θ of Ith cable segment in degrees
TENI(I)	Initial value of tension of Ith cable segment in pounds or Newtons
XPI(I), YPI(I), ZPI(I)	Initial values of ($\dot{x}$, $\dot{y}$, $\dot{z}$) of Ith node in feet/second or meters/second

DEFINITION OF INPUT VARIABLES FOR SURFACE BUOY, (FSM(1) > 2000)

XSI(I), YSI(I), ZSI(I) [2]	Initial values of (x, y, z) of buoy center in feet or meters
XPSI(I), YPSI(I), ZPSI(I)	Initial values of ($\dot{x}$, $\dot{y}$, $\dot{z}$) of buoy center in feet/second or meters/second
WFM	Magnitude of wind force in pounds
WFAD	Angle with x-axis of wind force in degrees
CDASBY	Drag area for y- direction in feet ^{2} or meters ^{2}
WASB	Weight in air in pounds or Newtons
XMSBV YMSBV	Virtual mass (mass + added mass) for motion in (horizontal, vertical) directions in slugs or kilograms
RTX,RTY, RTZ	Distance of cable attachment point from buoy center in (x, y, z) directions in feet or meters
DFTLIM	Limiting value of draft in feet or meters for which the program calculates the percent of time that the draft exceeds this value.

EXPLANATORY NOTES

1. For FSM(1) > 2000., the program accepts input data for a surface buoy. In this case, AXSM(K) is the submerged volume of the buoy in feet ^{3} or meters ^{3} at vertical distance AYSM(K) in feet or meters from the buoy center, defined as the origin of the local buoy coordinate system. Input data is entered starting with the bottom of the buoy and progressively moving up so that AYSM(1) is the largest algebraic value and AYSM(NSM) is the smallest algebraic value. Also, AYSM(1) - AYSM(NSM) = total buoy length L_S. Also, AZSM(K) is C_D A_{SH}, the submerged drag area in the horizontal direction in feet ^{2} or meters ^{2} at vertical distance AYSM(K) from the buoy center. The FIDSM(K) may take on any values, such as, say, 0.

For FSM(1) $< 0.$, the program calls the subroutine PRESC for the pre-scribed surface motions. This subroutine is used by the user to program pre-scribed motions which are not covered by Equations (9) and (10).

2. For FSM(1) $> 2000.$, the program calculates the draft H_S required to support the weight in air of the buoy, and the weight in fluid of the lower cables and bodies. The displacement YYSI of the buoy center from the ocean surface for this draft is given by

$$YYSI = H_S - AYSM(1) \tag{40}$$

For an input value of YSI $\leq 1.0 \times 10^{8}$, the program sets the initial value of YSI = YYSI. This feature is specially useful for those cases where the cable starts from an initially vertical position.

SAMPLE PROBLEMS

Two sample problems are given to illustrate usage and results calculated by the program.

FREE-FLOATING BUOY-CABLE PROBLEM

Steady Current

The first problem, shown in Figure 3, involves the deployment behavior of an initially vertical 100-foot (30.5 m) free-floating buoy-cable system, shown in Figure 3, in the presence of a current profile whose magnitude varies linearly from 2 knots at the surface to zero at y = 100 feet (30.5 m), as shown in Figure 3. Calculations were made for a series of variations of current direction α. Table 1 shows for each direction variation, the magnitude V_D and angle α_D of the final

steady-state drift velocity, the depth y_L and horizontal excursions x_L and z_L of the lower body measured from the surface buoy, the real time t_D required to reach the final steady-state configuration, and the ratio of execution time to real time, ET/RT. The table also shows, for comparison purposes, two sets of steady-state results calculated by the two-dimensional Program CABMOD. The first set is obtained by directly integrating the differential equations of equilibrium and iterating the drift velocity and buoy draft until the cable system is in equilibrium. This approach will yield more accurate steady-state results than the approach used in Program CAB3DYN since the cable is taken to be continuous rather than segmented. The second set of results calculated by Program CABMOD is obtained in a manner similar to that of Program CAB3DYN, where the cable is segmented and the deployment motion is calculated from an initially vertical position to the final steady-state shape.

The results shown in the table may be used to check the accuracy of the formulation and program logic in Program CAB3DYN in three ways. First, Cases 1 and 2 represent the same unidirectional current profile with reference direction shifted by 45 degrees. The table shows that the only difference in the calculated results for the two cases is a shift of 45 degrees in the drift velocity and horizontal displacements x_L and z_L. This shows that the calculation procedure for the horizontal x and z displacements are equivalent. Secondly, a comparison of Cases 1 and 6 shows that Program CAB3DYN predicts values of V_D and x_L which respectively differ by 2.7 and 0.4 percent from the exact steady-state formulation represented by Case 6. This comparison indicates the type of error incurred when the continuous cable is represented by four segments. Perhaps the most direct check is to compare Cases 1 and 7, since they represent the final steady-state results calculated by Programs CAB3DYN and

CABMOD, respectively, starting from identical initial states and identical segmented representations of the cable. Table 1 shows that the agreement is only to two significant places. Table 1 also shows that the CABMOD results for V_D and x_L respectively show oscillations of $\pm$ 0.01 ft/sec ($\pm$ 0.003 m/sec) and $\pm$ 0.1 ft ($\pm$ 0.03 m) about the average values. The reason for both of the above phenomena is that Program CABMOD models buoy pitch which still has not damped out at t = 40 sec, while Program CAB3DYN neglects all rotational modes of the buoy. The next problem gives a more valid comparison since the surface motion is identically modeled by both programs. The results do show that the presence of buoy pitch has relatively little effect on the average results.

Cases 3, 4, and 5 show results for three-dimensional configurations where the current changes in direction with depth. It is of interest to note that the effect of change in direction is to slow drift velocity by as much as 15% from the two-dimensional value given by Cases 1 and 2. Furthermore, the resultant horizontal displacement $\sqrt{x_L^2 + y_L^2}$ for the three-dimensional Case 5 is 20 percent larger than the two-dimensional value. Thus, the use of a unidirectional current need not predict the largest horizontal displacement.

The table shows that the time required to deploy from an initially vertical position to the final steady-state configuration is approximately 40 seconds for all cases. The ratio of execution time to real time is approximately 3.2 for Program CAB3DYN and 1.2 for Program CABMOD.

It is of interest to know the effect of cable added inertia, which is neglected in Program CAB3DYN, on the cable motions during deployment. The motions predicted by Programs CABMOD and CAB3DYN are not directly comparable due to to their previously noted difference in the modeling of the surface buoy.

However, it should be noted that Program CABMOD[2] can be run with and without cable added inertia by setting the input variable AMC equal to 1. and 0., respectively. Table 2 shows values of the velocities $\dot{x}$ at the cable nodes computed by Program CABMOD for AMC = 1 and AMC = 0 for different times t during deployment. In the table, Node 0 represents the point of attachment of the cable to the buoy while Node 4 represents the lower weight. The table also shows the ratio ET/RT for AMC = 1 and AMC = 0. The results show that cable added inertia has little effect on the motions since the maximum difference between the results for AMC = 1 and AMC = 0 is only 0.004 ft/sec (0.0012 m/sec). As expected, the largest differences occur at the beginning of the deployment when inertia effects are the largest. The table also shows that ET/RT is 1.2 for the AMC = 1 case compared to 1.1 for the AMC = 0 case.

Comments on Efficient Use of Program for Slowly Varying Current

The results contained in Table 1 offer suggestions on efficient ways of using Program CAB3DYN for the following problem which otherwise would require prohibitive amounts of computer time. Suppose it is of interest to solve the above problem with the following two differences. First, the current is no longer steady but exhibits a sinusoidal variation with time with a period of, say, three hours. Secondly, the shape of the cable is desired at 20 nodes instead of only 4 nodes. A straightforward use of the program would model the cable with 20 nodes and compute the motion of the cable over three hours. Table 1 indicates that execution time is approximately 3.2 times real time for 4 nodes. Reference 1

indicates that execution time is proportional to the square of the number of nodes NCAB

$$ET \propto (NCAB)^2 \tag{41}$$

Thus, the execution time ET required to calculate motions for three hours for 20 nodes would be approximately

$$ET \approx 3.2 \times \left(\frac{20}{4}\right)^2 \times 3 \approx 240 \text{ hours} \tag{42}$$

Assuming that such an amount of computer time were available, the cost using the lowest priority available at the Center, block time priority PO, would be approximately

$$\text{Cost} = 240 \quad \times \$125 = \$\ 30{,}000 \tag{43}$$

However, the results shown in Table 1 suggest alternate ways of obtaining the desired results at much lower computer cost. Table 1 shows that the cable requires only 40 seconds to deploy from an initial vertical state to the steady-state configuration for a given current profile. Since this is a much smaller time scale than the period of the current variation, the cable system at a given time is essentially in equilibrium with the instantaneous current profile. The table also shows that the steady-state drift velocity obtained by using only four nodes is only a few percent different from the exact value obtained by taking the cable to be continuous. These facts suggest the following alternate approach.

Use Program CAB3DYN with relatively few cable segments to calculate the steady-state drift velocity at, say, 20 equally spaced values of time t covering the current profile cycle. If the cable is started from a vertical position each time, deployment time would be approximately 40 seconds. On the other hand, if the more efficient method of using the steady-state values from the preceding time as the initial conditions are adopted, deployment time to reach the new steady-state configuration will be substantially reduced to perhaps a few seconds. The 20 calculated drift velocity vectors $\vec{V}_D$ may then be used to generate 20 resultant current profiles $\vec{c}_r$ given by

$$\vec{c}_r(y) = \vec{c}(y) - \vec{V}_D \tag{44}$$

These current profiles along with the calculated values of buoy draft may then be input into the three-dimensional steady-state Program CAB3E[4], which calculates the configuration at any number of positions along the cable at the cost of only a few seconds per case. Thus, even if one assumes that deployment time is 40 seconds for each current profile, total execution time ET for this alternate approach is given by

$$ET \approx 20 \times 40 = 800 \text{ sec} \tag{45}$$

The cost using overnight priority P2 would be approximately

$$\text{Cost} = 800 \text{ X } \$0.06 = \$48 \tag{46}$$

Thus, the computer time and cost are substantially lower than the straight-forward approach while still yielding results of comparable accuracy.

TOWING CABLE PROBLEM

The second problem, shown in Figure 4, involves the deployment behavior of an initially vertical 500- ft (152.4 m) cable towed at various speeds. Calculations were made for the cable with and without the neutrally buoyant array shown in Figure 4. The cable is modeled by 5 equal segments.

In addition to Programs CAB3DYN and CABMOD, Program CABTRAN[5] was also used to calculate the deployment motion. In this program, the lower array is essentially lumped with the last cable segment. In order to make the results calculated by Programs CAB3DYN and CABMOD equivalent to those calculated by Program CABTRAN, the array drag was accounted for by increasing the diameter and drag coefficient C_D of the last segment.

For cases of the cable towing the neutrally buoyant array, Programs CABMOD and CAB3DYN predict converged steady-state results for only the 1-knot case. Table 3 shows four sets of calculated results for the angle ϕ_V of each cable segment at various times t during the deployment for this case. The table shows results calculated by Program CABTRAN, Program CAB3DYN, and Program CABMOD with and without added cable inertia effects. At t = 1,500 sec, when the deployment is complete, the table also shows the steady-state cable angle calculated by Program CABMOD taking the cable to be continuous. Finally, the table also shows the ratio of execution time to real time for each of the programs.

The table shows that the cable angles ϕ_V calculated by Program CABMOD, with and without added cable inertia, and by Program CAB3DYN all agree to within 0.02 degrees during the entire deployment motion. As in the towing cable problem, the results show that cable added inertia effects, which are neglected in Program CAB3DYN, have little effect on the motion. The results also show that the formu-

lation and logic in Programs CABMOD and CAB3DYN for two-dimensional cases are essentially equivalent. The results for Problem 1 have already shown that the formulations in Program CAB3DYN for the x and z directions are equivalent.

The table shows that the cable angles predicted by Program CABTRAN for the upper segments agree with the CABMOD and CAB3DYN results to within two degrees during the entire deployment. However, there are large differences for the two lower segments. In particular, there are differences of 6 and 17 degrees at t = 1,500 seconds. The table shows that the CABMOD and CAB3DYN angles at t = 1,500 seconds agree to within three degrees with the exact steady-state results, where the cable is taken to be continuous. The principal error in the configuration calculated by Program CABTRAN is that there is too large a jump in the angle of the two lower segments which is opposite to the trend along the rest of the cable.

In view of this incorrect steady-state behavior, Program CABTRAN was exercised for a series of towing speeds and number of cable segments NCAB. Table 4 shows the steady-state cable angles, obtained at t = 1,500 seconds for towing speeds V_S = 1, 3, 5 and 10 knots and values of NCAB = 5, 10 and 20 segments. The table also lists the ratio ET/RT for each value of NCAB.

The table shows that for a given speed, the type of error does not change with number of segments. For the case of 1 knot, the previously observed large jump in angle between the two lower segments, which is opposite to the trend along the rest of the cable, occurs again for NCAB = 10 and 20. For $V_S \geq$ 3 knots, the type of error is somewhat different. For all three values of NCAB, the cable angle oscillates along the cable, with the largest oscillation occurring in the lower segments. The correct steady-state configuration would exhibit a monotonic variation

of angle along the cable.

For $V_S \geq$ 3 knots, Programs CAB3DYN and CABMOD are not able to predict converged steady-state solutions. The programs predict large oscillations in the inclination of the lowest segment, even at t = 2,000 seconds. The problem appears to be the shallow angle of the lowest segment, which contains the drag of the neutrally buoyant array. Computer runs for cases of no array and a heavy array do not exhibit the above instability behavior.

Due to the difficulties encountered by all three programs for the neutrally buoyant array, runs were made using Program CABMOD, without added cable inertia, and Program CABTRAN for the case of no array for several towing speeds V_S. Tables 5 and 6 show the cable angles calculated by both programs at various times during deployment for V_S = 1 and 5 knots, respectively. Program CABMOD was used instead of Program CAB3DYN due to its somewhat lower execution time, see Table 3. As shown in Table 3, the cable angles calculated by Program CABMOD agree to within 0.02 degree with corresponding values calculated by Program CAB3DYN.

Table 5 shows that for V_S = 1 knot, the calculated cable angles for all the segments agree to within 0.2 deg during the entire deployment. Table 6 shows that for V_S = 5 knots, the agreement is somewhat worse, with differences which are typically one to two degrees. Table 5 also shows that both programs predict some overshooting past the steady-state angles. For both towing speeds, the calculated steady-state angles, which correspond to a straight line, agree to within 0.01 deg.

SUMMARY

On the CDC 6700 computer, Program CAB3DYN requires approximately 41,000 octal words to load, 30,400 octal words to execute, and 18 seconds to compile.

For two sample problems involving the deployment of a free-floating buoy-cable and a towing cable, execution time ranges from 1.2 to 2.5 times the corresponding time required by the two-dimensional Program CABMOD. The ratio of execution time to real time ranges from 0.6 to 3.3.

For both problems, the results show that cable added inertia, which is neglected in Program CAB3DYN, has little effect on the motions during deployment. The accuracy of the formulation in Program CAB3DYN is verified by noting the agreement with Program CABMOD for a towing cable case, and the equivalence of the results calculated by Program CAB3DYN for two unidirectional currents.

The results calculated by Program CAB3DYN for the free-floating buoy-cable problem agree only in an average sense with those calculated by Program CABMOD. This is due to the presence of buoy pitch terms contained in Program CABMOD, which are neglected in Program CAB3DYN. It is shown that the steady-state results calculated by modeling the deployment motion with four cable segments differs by only a few percent from exact results obtained by considering the cable to be continuous. An efficient method is indicated for using Program CAB3DYN, in conjunction with the three-dimensional steady-state Program CAB3E, to calculate the motions and detailed shape of a free-floating cable system in the presence of a slowly varying current. The method essentially consists of using Program CAB3DYN with relatively few segments to calculate the steady-state draft of the surface buoy and current profile relative to the cable system at a series of given times during the current cycle. The draft and relative current profile are then read into Program CAB3E to calculate the cable configuration at any number of positions along the cable.

For cases where the cable is towing a neutrally buoyant array, Programs CABMOD and CAB3DYN predict unstable motions of the lower segment at higher towing speeds. This instability is apparently due to the shallow angle of the lower segment since it disappears for cases of no array and a heavy array. Program CABTRAN, which neglects all inertia effects, was also used to calculate results for the towing cable problem. This program predicts incorrect steady-state configurations for the case of the neutrally buoyant lower array. For cases of cable alone, the cable angles predicted by Programs CABMOD and CAB3DYN usually agree to within one or two degrees with those predicted by Program CABTRAN during the entire deployment cycle.

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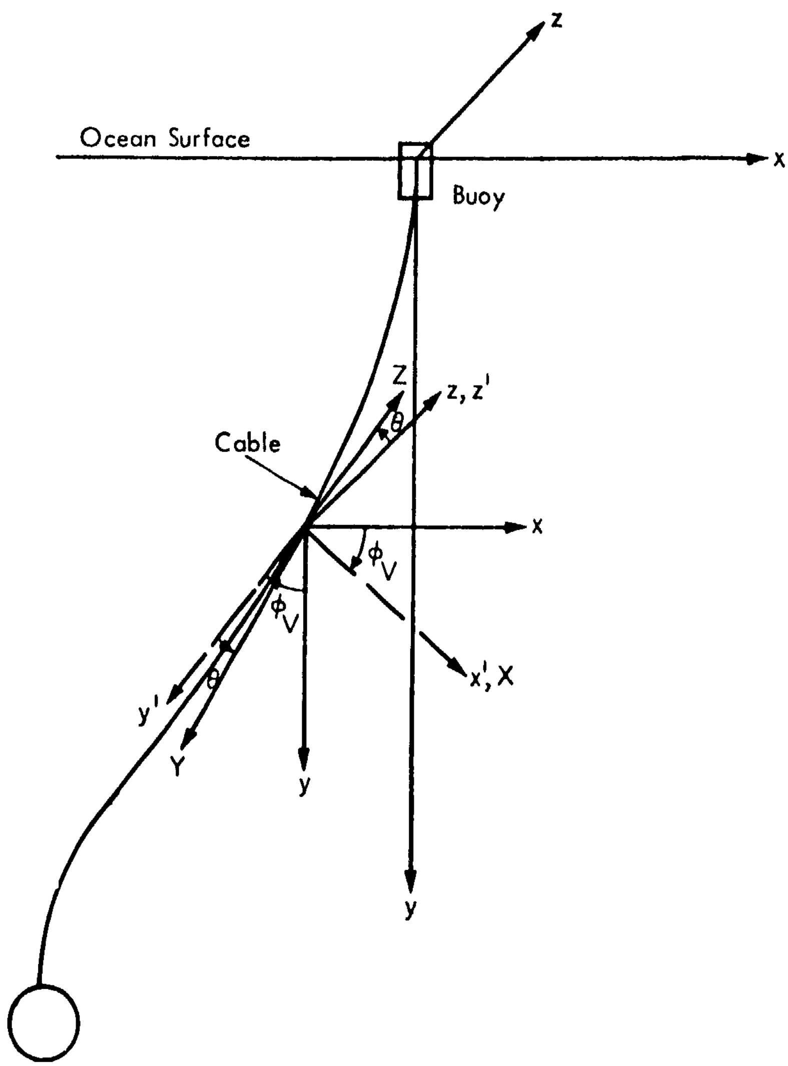


$\cos(X,x) = \cos\phi_V$ $\cos(X,y) = \sin\phi_V$ $\cos(X,z) = 0$

$\cos(Y,x) = -\cos\theta\sin\phi_V$ $\cos(Y,y) = \cos\theta\cos\phi_V$ $\cos(Y,z) = \sin\theta$

$\cos(Z,x) = \sin\theta\sin\phi_V$ $\cos(Z,y) = -\sin\theta\cos\phi_V$ $\cos(Z,z) = \cos\theta$

Figure 1 - Geometrical Configuration of Cable System

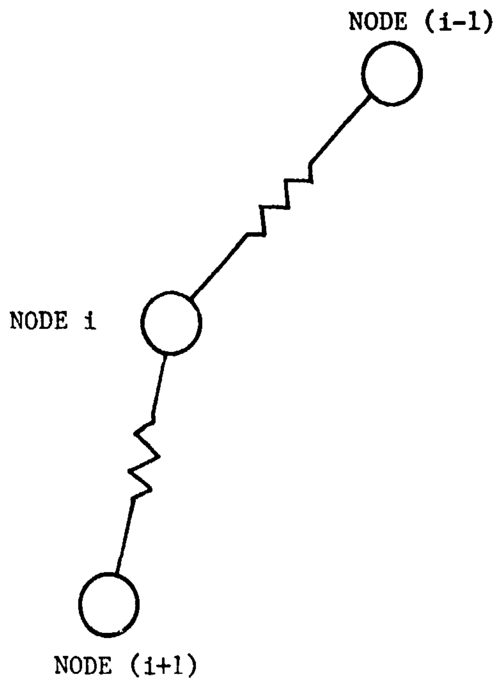
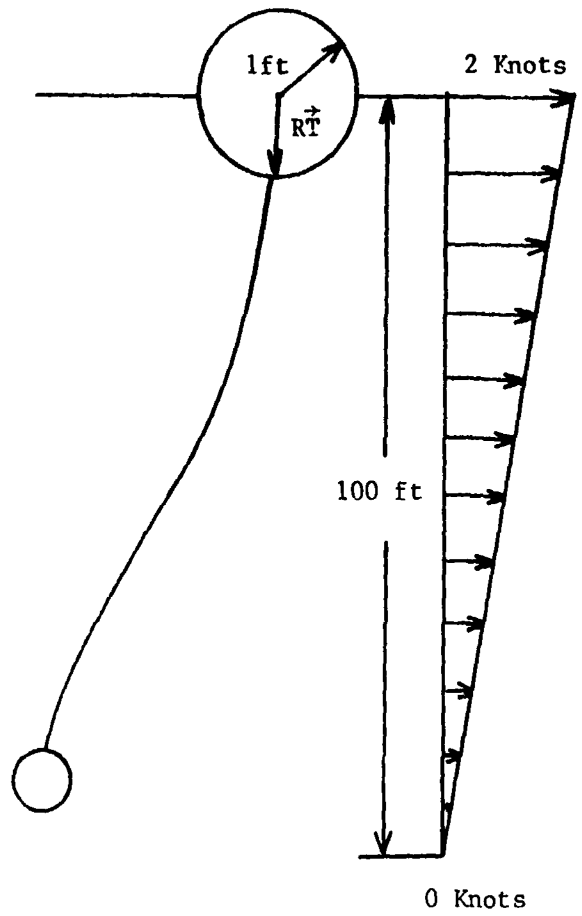


Figure 2 - Segmented Representation of Cable



Buoy Parameters:

$\vec{WF} = 0$

$C_D A_{Sy} = 0.79 \text{ ft}^2$

$W_{SAIR} = 100.0$ lb

$M_{SVH} = 5.19$ slugs

$M_{SVy} = 6.59$ slugs

$RT_x = RT_z = 0$

$RT_y = 1.0$ ft

$y = y_o$ (ft)	Vol (ft^3)	$C_D A_{SH}$ (ft^2)
AYSM	AXSM	AZSM
1.00	0.00	0.00
0.30	1.18	0.746
0.15	1.63	0.746
0.0	2.09	0.746
-0.15	2.56	0.746
-0.30	3.01	0.746
-1.00	4.19	1.00

Lower Body Parameters:

$\vec{BT} = 0$

$W_B = 20.0$ lb

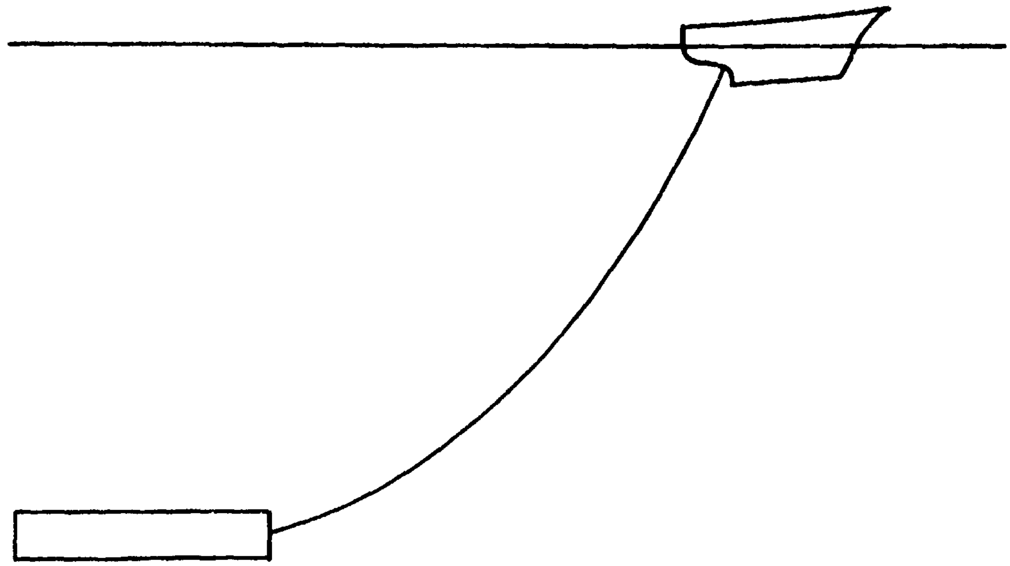
$C_D A_{BH} = 0.5 \text{ ft}^2$

$C_D A_{By} = 0.5 \text{ ft}^2$

Cable Parameters:

$L = 100$ ft	$C_T = 0.02$	$T_r = 0$ lb
$d = 0.12$ in	$W = 0.05$ lb/ft	$C1 = 2000$ lb
$C_D = 1.4$	$\mu = 0.02$ sl/ft	$NCAB = 4$

Figure 3 - Parameters for Free-Floating Buoy-Cable System of Sample Problem 1



Cable Parameters:

$L = 500$ ft	$W = 0.2$ lb/ft	NCAB = 5
$d = 0.5$ in	$\mu = 0.0089$ sl/ft	
$C_D = 1.8$	$T_r = 0$ lb	
$C_T = 0$	$Cl = 10000$ lb	

Lower Array:

$L = 200$ ft	$W = 0$ lb/ft
$d = 1.0$ in	$\mu = 0.0109$ sl/ft
$C_D = 1.8$	$T_r = 0$ lb
$C_T = 0$	$Cl = 10000$ lb

Figure 4 - Parameters for Cable Towing Lower Array of Sample Problem 2

TABLE 1 – FINAL STEADY-STATE RESULTS FOR FREE-FLOATING BUOY-CABLE SYSTEM

Case	Program	(deg) y=0	y=100	V_D (f/s)	(deg)	x_L (ft)	z_L (ft)	y_L (ft)	t_D (sec)	ET/RT
1	CAB3DYN	0	0	1.724	0	9.73	0	101.61	40	3.2
2	CAB3DYN	45	45	1.724	45	6.88	6.88	101.61	40	3.2
3	CAB3DYN	0	45	1.707	5.2	9.88	0.13	101.59	40	3.2
4	CAB3DYN	0	90	1.658	11.9	10.32	0.25	101.54	40	3.3
5	CAB3DYN	0	180	1.461	24.1	11.65	0.79	101.33	40	3.3
6	CABMOD*	0	0	1.772	0	9.77	0	101.61	--	---
7	CABMOD**	0	0	1.72	0	9.7	0	101.62	(40)	(1.2)
				$\pm$ 0.1		$\pm$ 0.1				

* Steady-state calculation for continuous cable

** Dynamic calculation for segmented cable, with and without cable inertia

TABLE 2 - VALUES OF VELOCITY $\dot{x}$ IN FT/SEC CALCULATED BY PROGRAM CAMBOD DURING DEPLOYMENT OF FREE-FLOATING BUOY-CABLE SYSTEM

	t = 0.2 sec		t = 0.6 sec		t = 1.0 sec		t = 2.0 sec	
NODE	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}
0	0.301	0.301	0.780	0.780	1.138	1.138	1.729	1.728
1	0.632	0.628	1.131	1.128	1.202	1.202	1.202	1.202
2	0.305	0.303	0.685	0.682	0.905	0.902	1.011	1.012
3	0.079	0.078	0.229	0.228	0.397	0.394	0.785	0.783
4	0.000	0.000	0.009	0.009	0.041	0.040	0.290	0.288

	t = 4.0 sec		t = 10.0 sec		t = 20.0 sec		t = 40.0 sec	
NODE	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}	CAB-MOD^{o}	CAB-MOD^{1}
0	2.115	2.115	1.895	1.895	1.731	1.731	1.712	1.712
1	1.695	1.694	1.816	1.815	1.726	1.726	1.713	1.713
2	1.312	1.312	1.719	1.717	1.720	1.72	1.714	1.714
3	1.010	1.008	1.639	1.638	1.716	1.716	1.715	1.715
4	0.974	0.974	1.595	1.596	1.712	1.712	1.716	1.716

CAMBOD^{o} ET/RT = 1.20
CAMBOD^{1} ET/RT = 1.10
CABMOD^{o} = CABMOD with no cable added inertia (AMC = 0)
CABMOD^{1} = CABMOD with cable added inertia (AMC = 1)

TABLE 3 - VALUES OF CABLE ANGLE ϕ_V IN DEG DURING DEPLOYMENT OF CABLE TOWING LOWER ARRAY AT 1 KNOT

	t = 10 sec				t = 20 sec			
Segm	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN
1	5.76	5.74	5.74	5.74	9.46	9.44	9.45	9.46
2	2.16	2.15	2.15	2.14	4.65	4.64	4.64	4.63
3	0.92	0.92	0.92	0.90	2.43	2.42	2.42	2.35
4	0.54	0.53	0.53	0.44	1.67	1.66	1.66	1.31
5	0.20	0.22	0.22	0.34	0.72	0.71	0.71	1.17

	t = 50 sec				t = 100 sec			
Segm	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN
1	16.84	16.83	16.84	16.86	24.40	24.39	24.40	24.28
2	11.34	11.33	11.33	11.25	19.87	19.86	19.86	19.46
3	7.75	7.74	7.74	7.34	16.49	16.48	16.48	15.23
4	6.57	6.55	6.55	5.01	16.59	16.57	16.57	12.24
5	3.27	3.26	3.26	5.42	9.83	9.82	9.82	15.78

	t = 200 sec				t = 500 sec			
Segm	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN	CAB-MOD^1	CAB-MOD^0	CAB-3DYN	CAB-TRAN
1	32.96	32.96	32.96	32.31	39.45	39.45	39.45	38.60
2	30.84	30.83	30.84	29.51	39.62	39.62	39.62	38.08
3	29.96	29.95	29.96	26.61	40.50	40.50	40.50	36.98
4	35.76	35.75	35.75	24.22	54.50	54.50	54.49	30.33
5	26.57	26.55	26.56	38.82	64.01	64.00	64.02	69.56

TABLE 3 - (continued)

t = 1,500 sec

Segm	CAB-MOD^{1}	CAB-MOD^{0}	CAB-3DYN	CAB-TRAN	CAB-MOD^{ss}
1	39.52	39.52	----	39.10	39.58
2	39.66	39.66	----	38.83	39.76
3	40.16	40.16	----	37.93	40.37
4	47.61	47.61	----	30.05	44.77
5	65.55	65.56	----	71.63	66.97

CABMOD^{1} ET/RT = 0.43

CABMOD^{0} ET/RT = 0.55

CAB3DYN ET/RT = 0.63

CABTRAN ET/RT = 0.01

CABMOD^{1} = CABMOD with added cable inertia

CABMOD^{0} = CABMOD with no added inertia

CABMOD^{ss} = Steady-state results calculated by CABMOD taking the cable to be continuous

TABLE 4 - STEADY-STATE VALUES OF CABLE ANGLE ϕ_V IN DEG CALCULATED BY PROGRAM CABTRAN FOR VARIOUS VALUES OF V_S AND NCAB

NCAB = 5 ET/RT = 0.01

V_S (knots)

Segm	1	3	5	10
1	39.10	73.51	82.76	87.59
2	38.83	67.53	73.88	81.08
3	37.93	79.16	85.11	87.99
4	30.05	61.04	71.91	80.79
5	71.63	83.83	86.29	88.15

NCAB = 10 ET/RT = 0.01

Segm	1	3	5	10
1	39.34	71.64	78.58	82.53
2	39.32	71.64	79.59	86.71
3	39.30	71.60	77.72	81.84
4	39.26	71.83	80.93	87.40
5	39.19	70.90	75.85	81.16
6	39.04	73.84	83.34	88.03
7	38.71	66.86	73.14	80.61
8	37.61	80.36	86.01	88.48
9	27.15	59.46	70.92	80.29
10	76.58	85.51	87.30	88.65

TABLE 4 (continued)

NCAB = 20 ET/RT = 0.04

V_S (knots)

Segm	1	3	5	10
1	39.38	71.64	78.92	84.30
2	39.38	71.64	78.92	84.65
3	39.38	71.64	78.92	84.16
4	39.38	71.64	78.92	84.84
5	39.38	71.64	78.92	83.91
6	39.37	71.64	78.92	85.15
7	39.37	71.64	78.91	83.54
8	39.36	71.64	78.95	85.60
9	39.36	71.64	78.84	83.01
10	39.35	71.64	79.10	86.20
11	39.33	71.64	78.54	82.35
12	39.31	71.64	79.66	86.93
13	39.29	71.60	77.59	81.59
14	39.24	71.86	81.15	87.68
15	39.16	70.80	75.53	80.86
16	38.99	74.10	83.79	88.37
17	38.61	66.33	72.57	80.25
18	37.34	81.28	86.69	88.85
19	24.67	58.25	70.17	79.91
20	80.34	86.77	88.06	89.03

TABLE 5 - VALUES OF CABLE ANGLE ϕ_V IN DEG DURING DEPLOYMENT OF CABLE TOWING NO ARRAY AT 1 KNOT

	t = 10 sec		t = 20 sec		t = 30 sec		t = 50 sec	
Segm	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN
1	5.74	5.74	9.44	9.46	12.33	12.37	16.79	16.85
2	2.15	2.14	4.62	4.63	6.98	6.99	11.19	11.22
3	0.90	0.90	2.34	2.33	3.94	3.94	7.24	7.26
4	0.43	0.42	1.27	1.24	2.30	2.27	4.68	4.67
5	0.31	0.20	0.78	0.65	1.37	1.27	2.91	2.83

	t = 100 sec		t = 200 sec		t = 500 sec		t = 1,500 sec	
Segm	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN
1	24.11	24.20	31.92	32.00	38.54	38.56	39.40	39.39
2	19.27	19.34	29.03	29.12	38.13	38.16	39.40	39.39
3	14.87	14.92	25.84	25.94	37.62	37.65	39.40	39.39
4	11.17	11.19	22.49	22.57	37.00	37.05	39.40	39.39
5	7.80	7.76	18.20	18.22	35.58	35.64	39.40	39.39

CABMOD^0 = CABMOD with no cable added inertia

TABLE 6 – VALUES OF CABLE ANGLE ϕ_V IN DEG DURING DEPLOYMENT OF CABLE TOWING NO ARRAY AT 5 KNOTS

	t = 10 sec		t = 20 sec		t = 30 sec		t = 50 sec	
Segm	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN
1	36.95	37.20	63.24	62.24	78.12	75.22	80.64	80.29
2	9.33	9.44	27.73	28.44	49.87	49.75	82.06	76.72
3	2.63	2.65	10.47	10.86	23.96	24.98	61.72	59.65
4	0.93	0.93	4.17	4.30	10.94	11.48	39.04	38.84
5	0.51	0.35	1.86	1.67	4.65	4.63	16.21	18.37

	t = 100 sec		t = 200 sec		t = 500 sec		t = 1,000	
Segm	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN	CAB-MOD^0	CAB-TRAN
1	78.54	78.11	79.63	79.61	78.92	78.91	78.92	78.92
2	79.83	80.88	77.06	77.88	78.95	78.94	78.92	78.92
3	75.74	73.55	78.43	77.38	78.89	78.92	78.92	78.92
4	102.72	90.48	80.51	79.35	78.89	78.89	78.92	78.92
5	76.92	75.62	79.70	79.50	78.92	78.91	78.92	78.92

CABMOD^0 = CABMOD with no cable added inertia

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